COOLING, MONOPOLES, AND VORTICES IN SU(2) LATTICE GAUGE THEORY

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We study monopoles and vortices in $SU(2)$ lattice gauge theory on a 24^4 lattice at $\beta = 2.50$. We find a value of fundamental string tension from monopoles in the maximum Abelian gauge consistent with the full $SU(2)$ value. Using direct and indirect center gauges, we find fundamental string tension values from P-vortices which are larger than the full $SU(2)$ result. After a single cooling sweep, the string tensions from monopoles and P-vortices are all 30% lower than the full $SU(2)$ value, while the $U(1)$ string tension in the maximum Abelian gauge remains consistent with the full $SU(2)$ result. Blocking the lattice after cooling does not restore the low values of string tension found with monopoles and P-vortices.

1 Introduction

The problem of understanding quark confinement in QCD is as old as QCD itself-even older, since there was evidence for quarks well before QCD was precisely formulated. Among physicists working to understand confinement, there is universal agreement that the essence of confinement can be addressed in the pure gauge theory, without light dynamical quarks. In addition, there is near-universal agreement that the mechanism of confinement will be ‘topological’ in nature, caused by a dense gas or network of topological objects which can disorder Wilson loops and produce a linear, confining quark potential.

Even without dynamical quarks, there are a host of quantities which a theory of confinement must explain. First and foremost is the heavy quark potential and in particular, the string tension in the fundamental representation. This has the most real-world relevance in the spectra of mesons composed of charmed quarks. Although only the $SU(3)$ color gauge group is relevant to the real world, the non-perturbative dynamics of all the $SU(N)$ theories appear to be quite similar, so as a preliminary to work on $SU(3)$, there has been a concerted effort to understand confinement for the simpler case of an $SU(2)$ gauge group.

The list of topological objects which are possibly relevant to confinement is short; instantons, monopoles, and vortices. Although instantons have a firm basis as semiclassical objects in the continuum limit, recent work casts serious doubt on them as agents of confinement. [1] Accepting this conclusion leaves monopoles and vortices. The monopole and vortex approaches to confinement share a common postulate: namely that the long-range confining physics should be Abelian in character. They differ in which Abelian subgroup of $SU(N)$ is postulated to carry the confining physics.

The path leading to monopoles is normally called 'Abelian projection' , in which the projection $SU(N) \rightarrow U(1)^{N-1}$ takes place after gauge-fixing. Physical quantities may then be calculated using the projected $U(1)$ fields; this is called 'Abelian dominance', or a further projection made, in which $N-1$ species of magnetic currents of monopoles are located and then physical quantities calculated. This latter is called 'monopole dominance'.

The other topological approach, that leading to vortices, makes the projection $SU(N) \rightarrow Z(N)$, where $Z(N)$ is the center of $SU(N)$. There are two methods of proceeding. The one most analogous to Abelian projection is called 'center projection', and uses gauge-fixing followed by a projection of $SU(N)$ links to $Z(N)$ links. [2,3] These $Z(N)$ links are then used to calculate physical quantities. Vortices are associated with plaquettes which are pierced by non-vanishing $Z(N)$ flux. Here, there is no distinction like that between Abelian dominance and monopole dominance, since every $Z(N)$ Wilson loop can be expressed as a product of plaquettes over a surface which spans the loop. The second approach to vortices treats a Wilson loop as a 'vortex counter'. To calculate the heavy quark potential, the Wilson loop is then simply replaced by its $Z(N)$ part, which is the sign of the trace for $SU(2)$. This vortex part then carries all the information about confinement. [4] To distinguish these two approaches to confinement via vortices, we will refer to vortices located by examining plaquettes after gauge-fixing and center projection as 'P-vortices', and simply use the term 'vortices' if no gauge-fixing or projection at the one-link level is used. The ideal situation would be that a P-vortex is locating the geometrical center of an actual physical vortex, so in that case results from the two methods would agree. However, calculations done previously and in this work show that this ideal picture is too naive.

The present work is devoted to problems with monopoles and vortices which arise when configurations are smoothed by cooling. We restrict ourselves to a discussion of the fundamental string tension for the case of an $SU(2)$ gauge group.

2 Gauge-fixing

The use of gauge-fixing to locate topological objects is common to both the Abelian and center projection methods. Since projection actually deletes certain dynamical degrees of freedom, the use of a particular gauge here is different than say an nth order perturbative calculation of a gauge invariant quantity. There, if all terms of a given order are calculated, one gauge may be more or less convenient than another, but all will give the same answer in the end. On the other hand if, as in Abelian and center projection, certain dynamical degrees of freedom are removed after gauge-fixing, the resulting estimate of a physical quantity like the string tension may depend on the gauge condition used. Thus even though the quantity being calculated is gauge-invariant in the full theory, under Abelian or center projection, there may be an optimal or 'maximum' gauge which produces the best approximation to the desired physical quantity. Further, the gauges commonly used in Abelian and center projection involve finding stationary points of gauge-functionals, and are subject to one form or other of the Gribov ambiguity. The question of how results depend on this ambiguity is important, but will not be pursued here. We use the gauge conditions which have been most successful in previous calculations, and gauge-fix each configuration once, i.e. one Gribov copy/configuration is retained.

2.1 Maximum Abelian Gauge

The maximum Abelian gauge (MAG) will be used for Abelian projection in the present work. Formulated in the continuum for $SU(2)$, we seek a minimum over gauge transformations of the functional

$$G_{mag} = \int [(A^1_\mu)^2 + (A^2_\mu)^2] d^4x, \tag{1}$$

which leads to the following differential condition:

$$(i\partial_\mu \pm eA^3_\mu)A^\pm_\mu = 0. \tag{2}$$

The lattice equivalent of minimizing G_{mag} is maximizing the functional G_{lmag} given by

$$G_{lmag} = \sum_{x,\mu} \frac{tr}{2} \left[U^\dagger_\mu(x)\sigma_3 U_\mu(x)\sigma_3 \right]. \tag{3}$$

The numerical implementation of the MAG involves a certain stopping criterion. We used the same criterion as in our previous work. [5] Expanding the gauge-fixed link U_μ in Pauli matrices,

$$U_\mu = U_\mu^0 + i\sum_{k=1}^{3} U_\mu^k \cdot \sigma_k, \tag{4}$$

we perform the lattice Abelian projection by extracting the U(1) link angle ϕ_μ^3

$$\phi_\mu^3 = 2\ \arctan(U_\mu^3/U_\mu^0). \tag{5}$$

Keeping only the $U(1)$ link formed from $\phi_\mu^3 = A_\mu^3 a$ is equivalent to retaining only the Abelian field A_μ^3 in the continuum.

The motivation for making the Abelian projection is really monopoles. For a $d = 3$ 't Hooft-Polyakov monopole in MAG, the 'charged' fields $A_\mu^{1,2}$ are short-ranged, and the Abelian field A_μ^3 is long-ranged and resembles that of a Dirac monopole with two Dirac units of charge, $eg = 4\pi$. The Dirac string that appears in this gauge is the basis of the Toussaint-DeGrand method of monopole location on the lattice. [6] In $d = 3$, a monopole is at the end of a Dirac string, while in $d = 4$ the magnetic current of a monopole lies on the edge of a Dirac sheet.

2.2 Direct and Indirect Center Gauges

In what is called the direct center Gauge (DCG), the following functional is maximized over gauge transformations:

$$G_{dcg} = \sum_{x,\mu} (tr(U_\mu(x)))^2. \tag{6}$$

Using the relation between the trace of group element matrices in fundamental and adjoint representations,

$$tr(U_A) = (tr(U_F))^2 - 1, \tag{7}$$

we see that DCG condition maximizes the trace in the adjoint representation. For small gauge fields this is the same as minimizing

$$\sum_{x,\mu} (A_\mu^a)^2, \tag{8}$$

which is the Landau gauge condition. The functional G_{dcg} in Eq.(6) is 'center-blind' , i.e. invariant to $U_\mu \rightarrow Z \cdot U_\mu$, where Z is a member of the center group, so the DCG can be thought of as a center-blind Landau gauge. All components of A^a_μ are suppressed as much as possible, modulo a center factor in the fundamental representation links.

A variant on DCG is the indirect center Gauge (ICG), where after a preliminary gauge-fixing to MAG, the functional

$$G_{icg} = \sum_{x,\mu} (\cos(\phi^3_\mu))^2 \tag{9}$$

is maximized over $U(1)$ gauge transformations. Having suppressed $A^{1,2}_\mu$ by the use of MAG, this is a center-blind way to finally suppress A^3_μ.

For both DCG and ICG, the center projection is done by writing

$$U_\mu(x) = sign(tr(U_\mu(x))) \cdot \bar{U}_\mu(x). \tag{10}$$

Non-perturbative, confining physics is postulated to reside in the $Z(2)$ gauge field $Z_\mu = sign(tr(U_\mu))$. The presence of a P-vortex is signified by a negative $Z(2)$ plaquette which is supposed to represent the physical center of an actual vortex. In our numerical calculations, the stopping criteria used for DCG and ICG were of a similar nature and quality to that used for MAG.

3 Unsmoothed Results

The calculations presented here are for the Wilson form of $SU(2)$ lattice gauge theory, at $\beta = 2.50$, on a 24^4 lattice. We have analyzed 49 configurations in the MAG, and 30 in DCG and ICG. In these configurations, we have extracted heavy quark potentials, and examined some features of the distribution of monopoles and P-vortices. As mentioned, we take one Gribov copy/configuration. The number of configurations is moderate, but as seen in Fig.(1), the potentials are extremely linear and noise-free, a characteristic of calculations with topological objects. The string tension in the fundamental representation is easily extracted from the slopes of the potentials vs R. The results are tabulated in Table 1. The corresponding full $SU(2)$ fundamental string tension at $\beta = 2.50$ is 0.033(2) from our own previous work [5] on 16^4, or 0.0325(12) from Bali, *et al* [7] on 32^4, at $\beta = 2.5115$. The figures in Table 1 show that the monopole string tension is very consistent with the full $SU(2)$ results, but that the P-vortex string tensions are too large, by an amount outside error bars. A high value of σ_{dcg} and σ_{icg} for one Gribov copy/configuration was also recently observed by Bornyakov *et al* . [8]

Turning to the distribution of monopoles and P-vortices, the percentage of links with magnetic current is 1.36(1), which means there are $\sim 18,000$ links with magnetic current in a typical configuration. The largest cluster has an

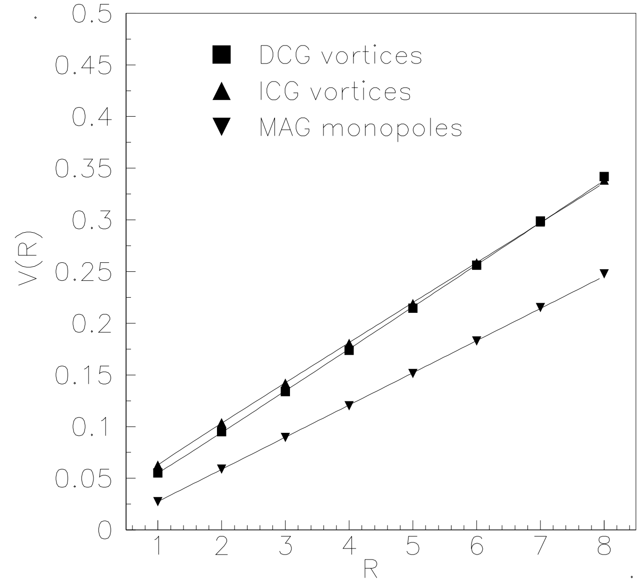

Figure 1: Heavy quark potentials from monopoles (MAG), and vortices (DCG and ICG).

Table 1: $\beta = 2.5$, 24^4 Wilson Action String Tensions

$\sigma_{mag}(mon)$	σ_{dcg}	σ_{icg}
0.031(1)	0.040(1)	0.039(1)

average size of 7554(124) links. Only the latter is relevant to confinement so really only $\sim 0.57\%$ of the links on the lattice play a role in the confining part of the magnetic current.

For P-vortices, the percentage of links pierced by a P-vortex is 3.21(1) for DCG and 3.78(1) for ICG. In the indirect center gauge, there is a strong correlation between the locations of magnetic current and of P-vortices. The magnetic current m_μ resides on the dual of the original lattice, so the timelike magnetic current m_t may be placed at the center of a spacial (xyz) cube of the original lattice, m_x at the center of a yzt cube, etc for m_y, m_z. As first noted by Greensite *et al* [9], a large percentage of the time when a 3-cube contains magnetic current, two of its faces are pierced by P-vortices. We measured this percentage and find 93(1)% for this result, consistent with Greensite *et al.* [9]

Table 2: Cooled Wilson action string tensions

$\sigma_{mag}(mon)$	σ_{dcg}	σ_{icg}
0.021(1)	0.021(1)	0.022(1)

In other words, in the ICG, P-vortices end on monopoles. Since P-vortices *are* visible, i.e. they cause minus signs in fundamental Wilson loops, they resemble Dirac strings (or sheets in $d = 4$) with 2π rather than 4π units of flux. A fundamental quark in a Wilson loop acts like a half-integer charge, so a vortex threading the loop gives a factor $\exp(i2\pi/2) = -1$. Now 'clock' or Z(n) approximations to $U(1)$ are often successfully used with rather large values of n. The ICG is an extreme form of this approximation where $U(1)$ is projected to $Z(2)$. The Coulomb flux of a monopole gets squeezed into a $Z(2)$ vortex passing through the monopole. This shows up on the lattice as two negative $Z(2)$ plaquettes on the cube-faces surrounding the monopole.

4 Smoothed Results

Not all the information contained in a sequence of configurations generated in a Wilson action simulation is relevant to the physics of confinement. Local smoothing of configurations is a way to suppress ultraviolet fluctuations, while keeping the long-range physics intact. In the present work, our smoothing operation is a single cooling sweep of the lattice, in which each link is replaced by its action-minimizing value in the fixed environment of nearby links or 'staples'.

The string tension is stable under a single cooling, showing that the cooled configurations still encode the information about confinement present in the original configurations. It is reasonable to demand that a description of the confining degrees of freedom also be stable under cooling. To test this we subjected the once-cooled configurations to the same gauge-fixing and object location procedures used in the previous section. The results are shown in Table 2.

The string tensions from monopoles, and vortices from DCG and ICG now agree, but all are $\sim 30\%$ low, compared to the full $SU(2)$ result. This poses a serious problem for claims that monopoles found after gauge-fixing to the MAG, or P-vortices found after gauge-fixing to either DCG or ICG are a correct identification of the infrared confining degrees of freedom.

It is of interest to see how the numbers and distribution of monopoles and P-vortices are affected by cooling. In DCG there are now only 1.7%

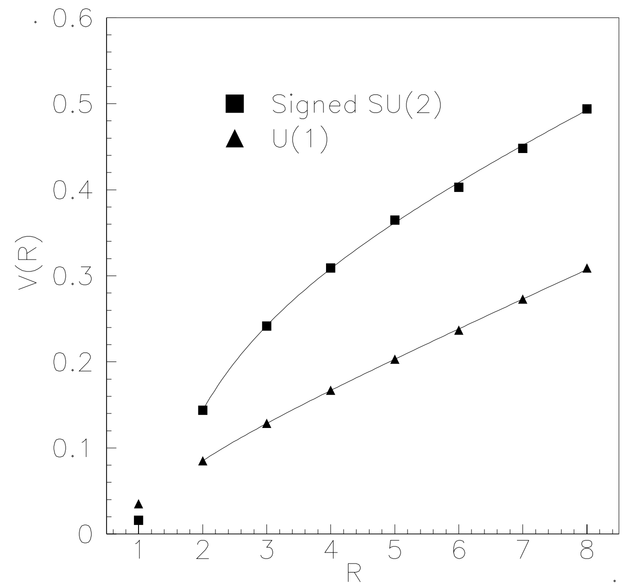

Figure 2: Potentials after cooling from $U(1)$ (MAG) and sign of Wilson loop

of plaquettes pierced by a P-vortex, an almost 50% reduction compared to uncooled configurations. There is a similar reduction of the number of P-vortices in ICG.

For monopoles deduced from the MAG, the reduction in number after cooling is even more dramatic-now only 0.16% of the links carry magnetic current, a reduction by a factor of ~ 8.4. Whereas before cooling the largest cluster of magnetic current contained ~ 7500 links, after cooling the largest cluster contains only ~ 1200 links. While 30% of the string tension has been lost, from another viewpoint, it is remarkable that 70% of the string tension can be obtained with a magnetic current occupying only 0.16% of the links of the lattice. Cooling also heavily suppresses small clusters, which are known to be irrelevant for confinement. The connection between P-vortices in ICG and MAG monopoles is even tighter; now 98(1)% of the time a monopole cube-face is pierced by two P-vortices.

The results just discussed show that the most straightforward application of gauge-fixing and object location methods are not stable under smoothing. It does not follow that all hope of a topological description of confinement is lost. From the viewpoint of vortices, we may use the second method of proceeding with vortices. This uses no gauge-fixing, instead the potential is calculated by replacing the Wilson loop by its sign, loop by loop. This yields a fundamental string tension of 0.031(1), consistent with the full $SU(2)$ result. From the viewpoint of Abelian projection, we may gauge-fix to MAG, and calculate the potential from the $U(1)$ links directly, without a further reduction to monopoles. This yields a string tension after cooling of 0.034(1), slightly high, but still consistent with the full $SU(2)$ numbers. The potentials from these two calculations are shown in Fig. (2). (The differing short range or Coulombic terms in the two potentials is easily explained by the fact that perturbative exchange of charged gluons is suppressed in the MAG.)

Putting these last two results together suggests that the problem lies neither with the idea of a topological explanation of confinement nor the use of gauge-fixing, but rather with our methods of location of topological objects.

5 Extended Objects

As just discussed, the $U(1)$ field obtained via Abelian projection on cooled lattices retains the full $SU(2)$ string tension. The usual expectation is that the confining part of such a $U(1)$ field can be characterized in terms of the magnetic current of monopoles. However, our attempts to find this current have failed so far. In addition to the standard approach described in Sec. 4, we have made two other attempts, both looking for monopoles on a larger scale. The first method begins by casting the cooled $SU(2)$ configurations into the MAG as before, but then looking for 'extended' monopoles by applying the Toussaint-DeGrand method to 2-cubes, rather than 1-cubes. [10] A second attempt was to block the cooled $SU(2)$ configurations in a standard way [10], gauge-fix to the MAG on the blocked lattice, followed by monopole location, also on the blocked lattice. In both attempts, the resulting monopole string tension is still $\sim 30\%$ lower than the corresponding full $SU(2)$ string tension. (Our previous work [10] on a 20^4 lattice held out some hope that blocking would restore the lost string tension, but the present work on 24^4 does not support this.)

Likewise, although the signed Wilson loops carry the full $SU(2)$ string tension, we have been unable to characterize these signed loops in terms of P-vortices. We have also applied blocking here, taking the cooled, blocked $SU(2)$ configurations into DCG and ICG. The resulting P-vortex string tensions are again $\sim 30\%$ low.

To summarize, on the positive side, we have shown that the maximum Abelian gauge and Abelian projection itself survive cooling, as does the method of dealing with vortices which does not try to pin down their locations. However, we have also shown, on a suitably large lattice, that present methods of locating topological objects are unstable against smoothing. This is a serious problem for claims that these objects correctly identify the confining degrees of freedom in $SU(2)$ lattice gauge theory.

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